Market Power in Hydropower Systems

by

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Background

- The use of market power is a potential problem of the deregulated electricity sector
- Hydro power is a special case due to zero variable cost, high effect capacity and water storage

Plan of the presentation

- The basic economic characteristics of hydro power
  - Social optimum
  - Monopoly
- The role of constraints
- A mixed hydro and thermal system

The basic hydro model

$$Max \sum_{t \in T} U_t(e_t^H) \ s.t. \sum_{t \in T} e_t^H \leq W$$

- The first order conditions

$$U_t'(e_t^H) \equiv p_t(e_t^H) = \lambda \, \forall t \in T$$

- The law of one price:
  Arbitrage and Hotelling's rule

Monopoly

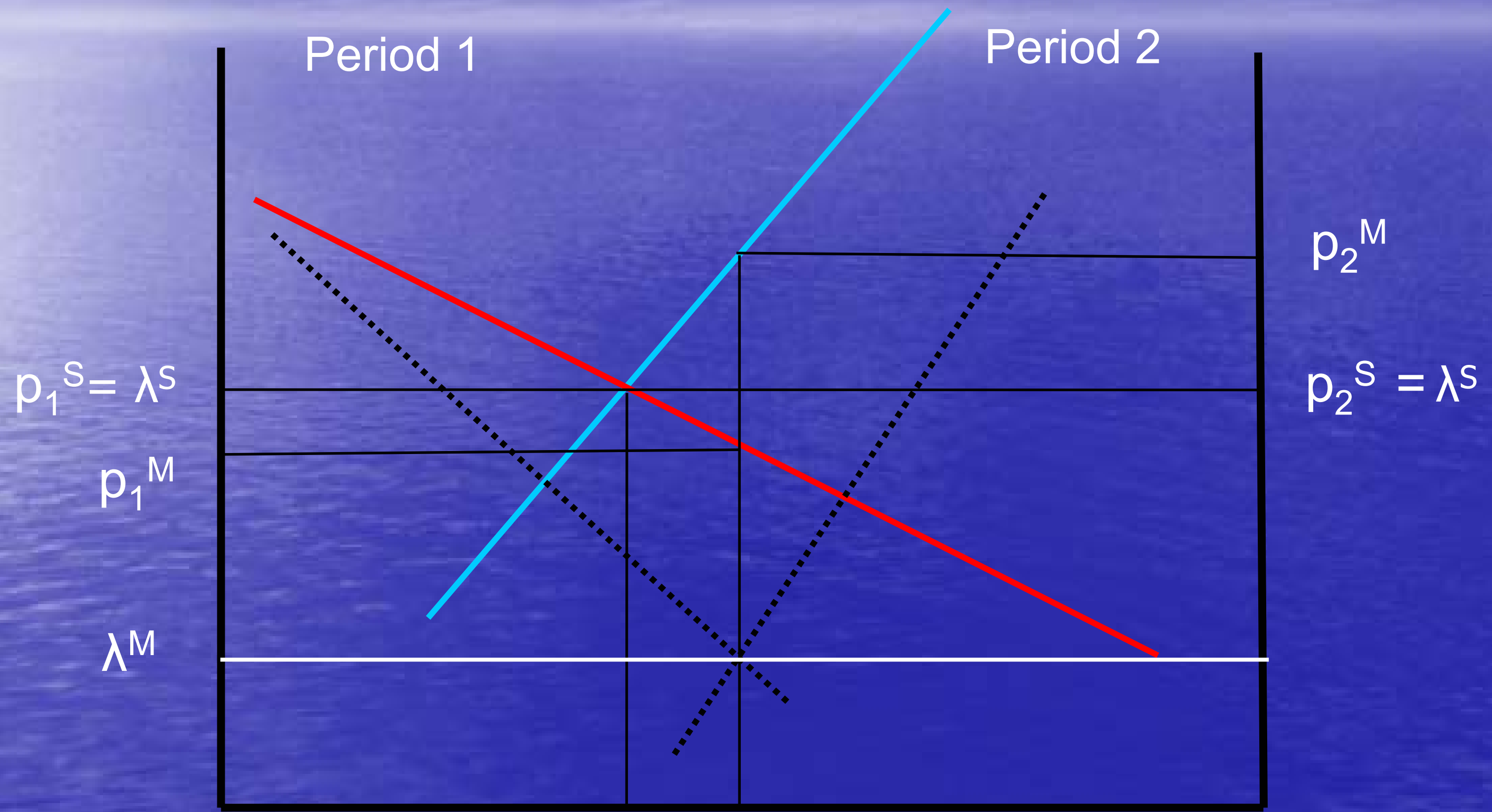
$$Max \sum_{t\in T} p_t(e_t^H)e_t^H \quad s.t. \sum_{t\in T} e_t^H \leq W$$

- Necessary condition

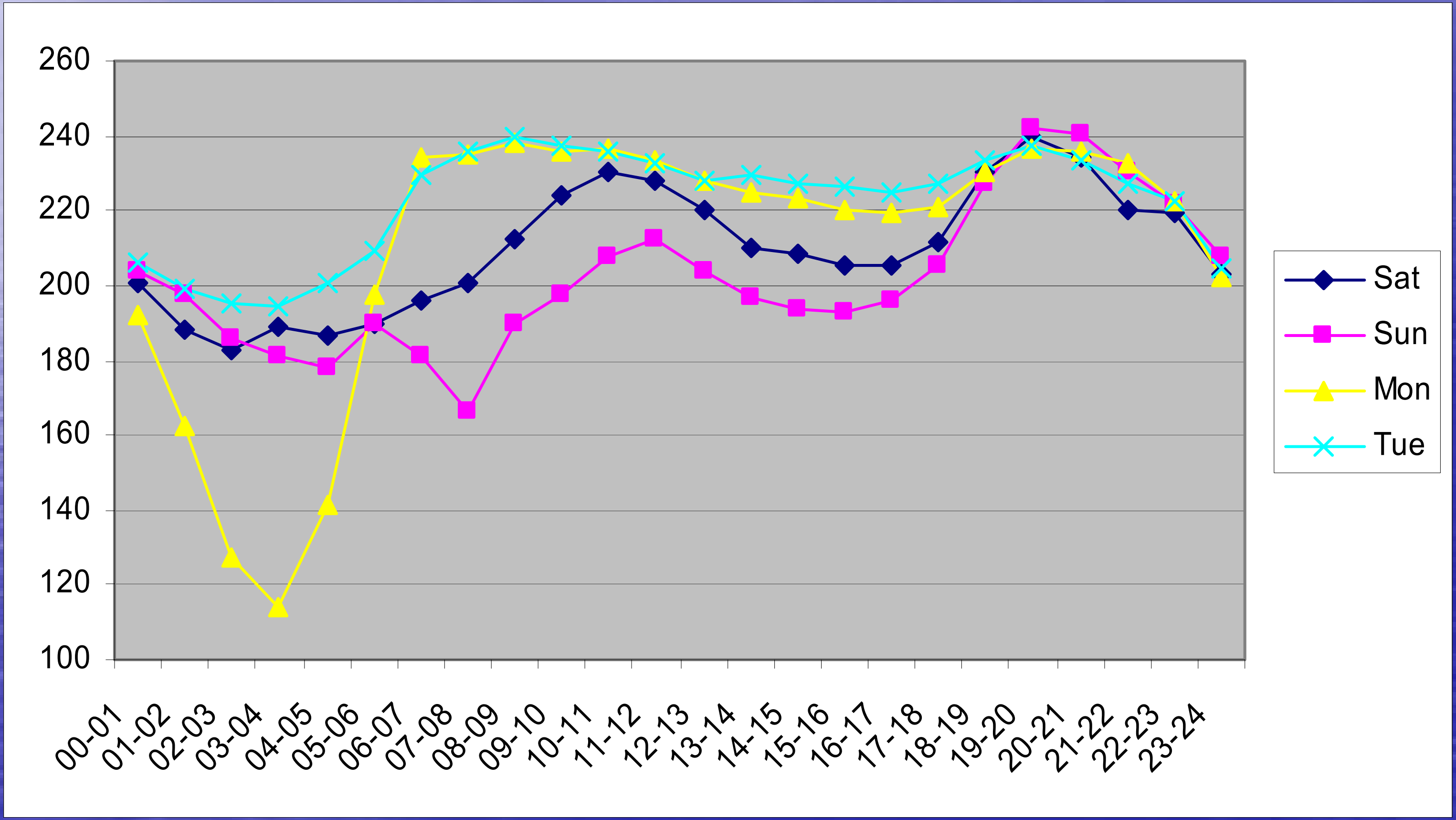
$$MR_t \equiv p_t(1+\breve{\eta}_t) = \lambda \forall t \in T$$

- The law of variable price:
- Prices differ between periods according to price flexibility, $\breve{\eta}_t$

Social optimum, and monopoly



Spot market prices Nord Pool 2-5/10/04



Constraints in a hydro system

- Must take flows w_t^{river}
- Reservoir size $R_t \leq \overline{R}$
- Minimum level $R_t \geq \underline{R}$
- Effect capacity $e_t^H \leq \overline{e}^H$
- Transmission capacity $e_t^H \leq \overline{e}_t^H$
- Water flows $\underline{e}_t^H \leq e_t^H \leq \overline{e}_t^H$
- Ramping up $e_t^H - e_{t-1}^H \leq r_t^u$
- Ramping down $e_{t-1}^H - e_t^H \leq r_t^d$

Social optimum and reservoir constraints

$$Max \sum_{t \in T} U_t(e_t^H)$$

$$s.t.$$

$$R_t \leq R_{t-1} + w_t - e_t^H, R_t \leq \bar{R}, t \in T,$$

Optimality conditions

$$U_t'(e_t^H) \equiv p_t(e_t^H) = \lambda_t, t \in T$$

$$-\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \perp R_t \geq 0, t \in T$$

- Bellman's principle: start with period T

$$\lambda_{T+1} = 0, \gamma_T = 0 \Rightarrow \lambda_T > 0 \; for \; R_T = 0$$

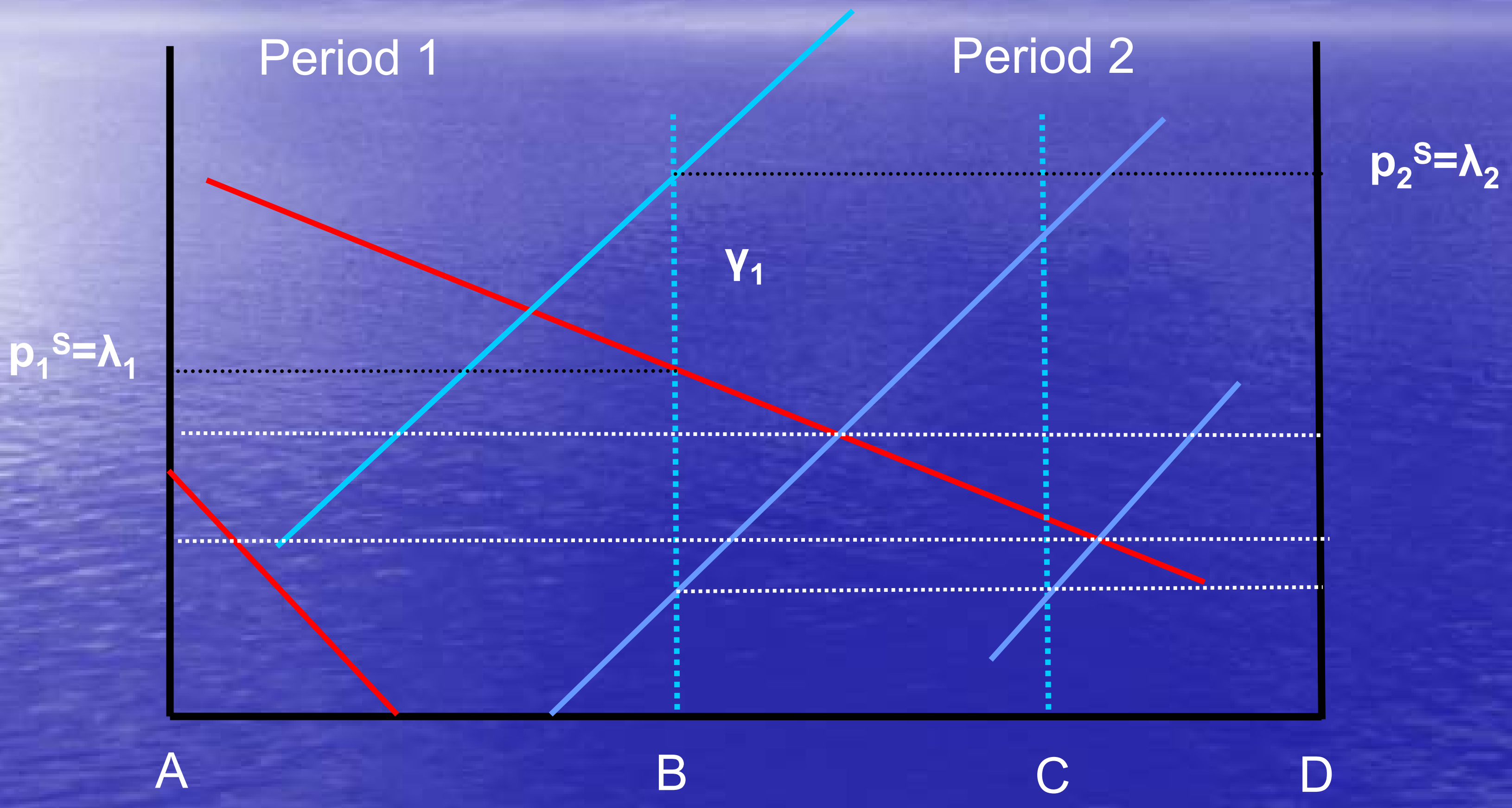
- Scarcity in period t<T and T:

$$\lambda_{t+1} = \lambda_T > 0, R_t = 0, \gamma_t = 0 \Rightarrow \lambda_t > \lambda_T$$

- Threat of overflow/overflow in period s<T

$$\lambda_{s+1} = \lambda_T > 0, R_s = \overline{R}, \lambda_T \geq \gamma_s > 0 \Rightarrow \lambda_s = \lambda_T - \gamma_s$$

Illustration of social optimum for two periods



Monopoly and reservoir constraints

$$Max \sum_{t \in T} p_t(e_t^H) e_t^H$$

$$s.t.$$

$$R_t \leq R_{t-1} + w_t - e_t^H, R_t \leq \bar{R}, t \in T$$

Monopoly solution

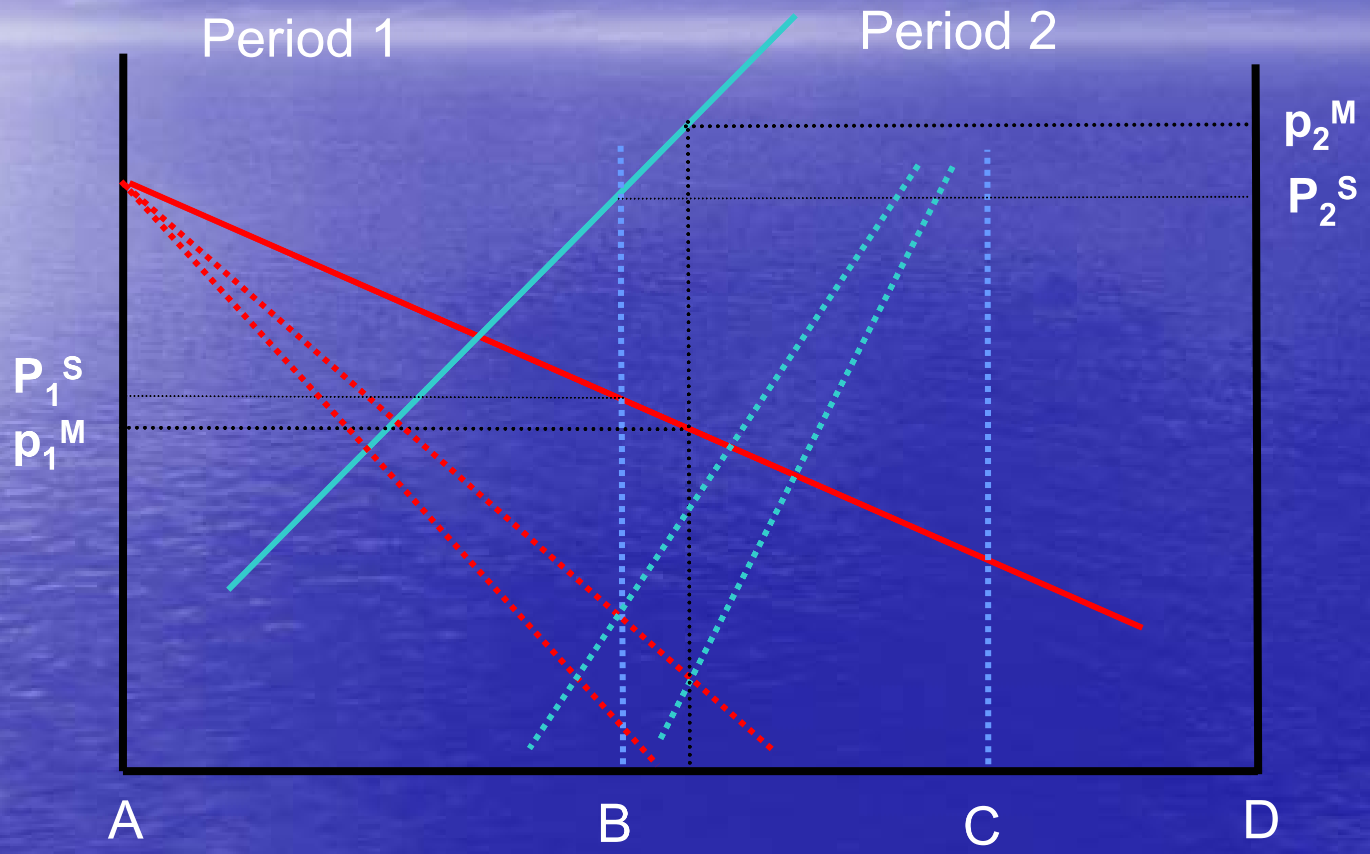
- Marginal revenue replace marginal willingness to pay

$$MR_t \equiv p_t(e_t^H)(1+\breve{\eta}_t) = \lambda_t, t \in T$$

$$-\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \perp R_t \geq 0 \ \forall t \in T$$

- Law of variable price
- May have spillage
- May not fill up reservoirs

Illustration of monopoly solution
Reservoir not filled up



Thermal capacity

- Individual thermal plants

$$c_{it} = c_i(e_{it}^T), e_{it}^T \le \overline{e}_i^T, i \in N$$

- Aggregation to thermal sector; merit order

$$Min \sum_{i \in N} c_i(e_{it}^T)(c_i' > 0)\, s.t.$$

$$\sum_{i \in N} e_{it}^T \ge e_t^T,\ e_{it}^T \le \overline{e}_i^T \Rightarrow$$

$$c_t = c(e_t^T), e_t^T \le \sum_{i \in N} \overline{e}_i^T = \overline{e}^T$$

Cooperative solution

$$Max \left\{ \sum_{t \in T} [(U_t(x_t) - c(e_t^T)] \right\} s.t.$$

$$x_t = e_t^H + e_t^T, \; \sum_{t \in T} e_t^H \leq W, \; e_t^T \leq \overline{e}^T$$

- Necessary conditions

$$p_t(x_t) - \nu_t \leq 0 \perp x_t \geq 0$$

$$\nu_t - \lambda \leq 0 \perp e_t^H \geq 0$$

$$-c'(e_t^T) + \nu_t - \theta_t \leq 0 \perp e_t^T \geq 0$$

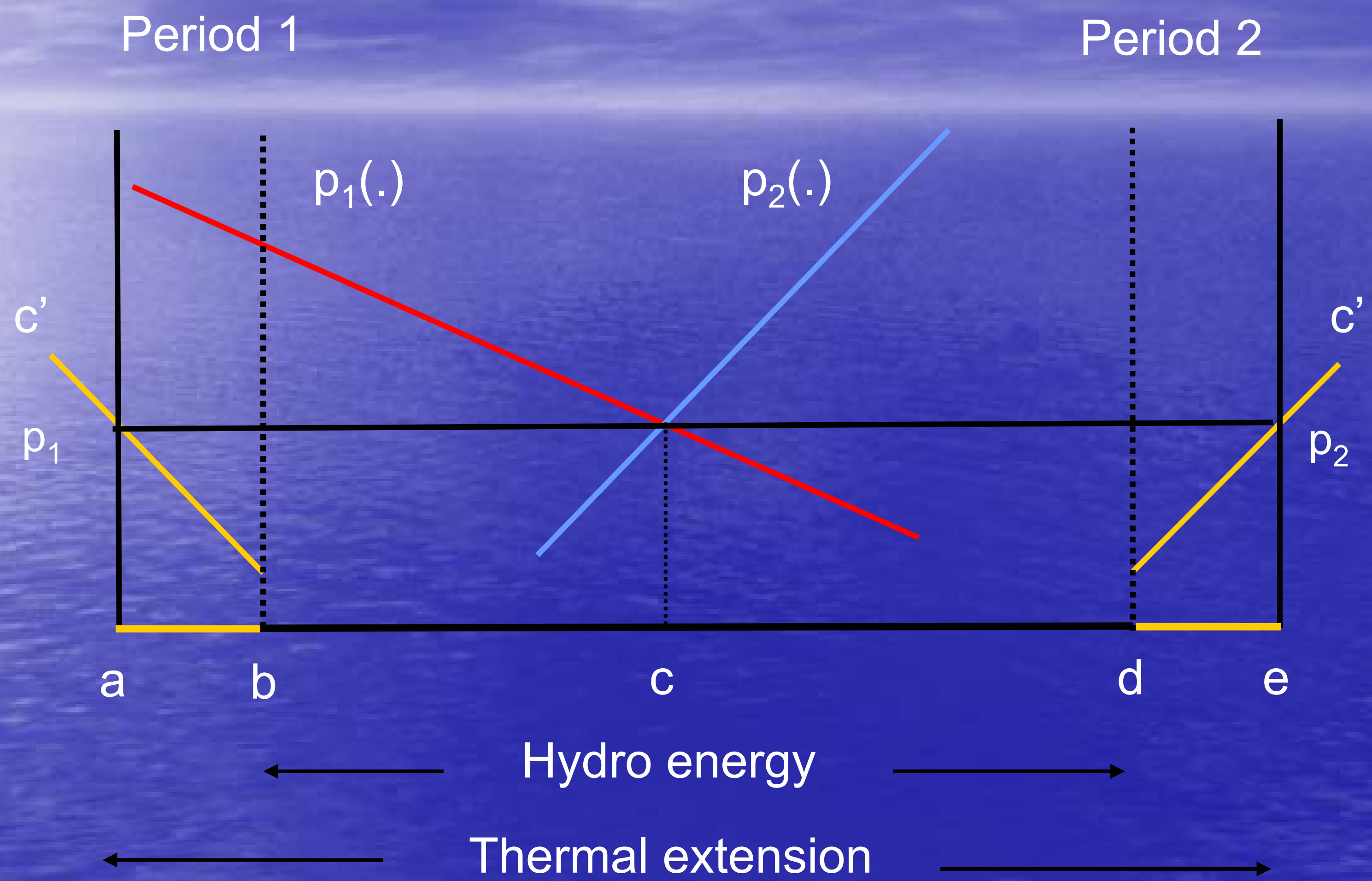
Cooperative solution, cont.

- Thermal as base load: constant load for all periods

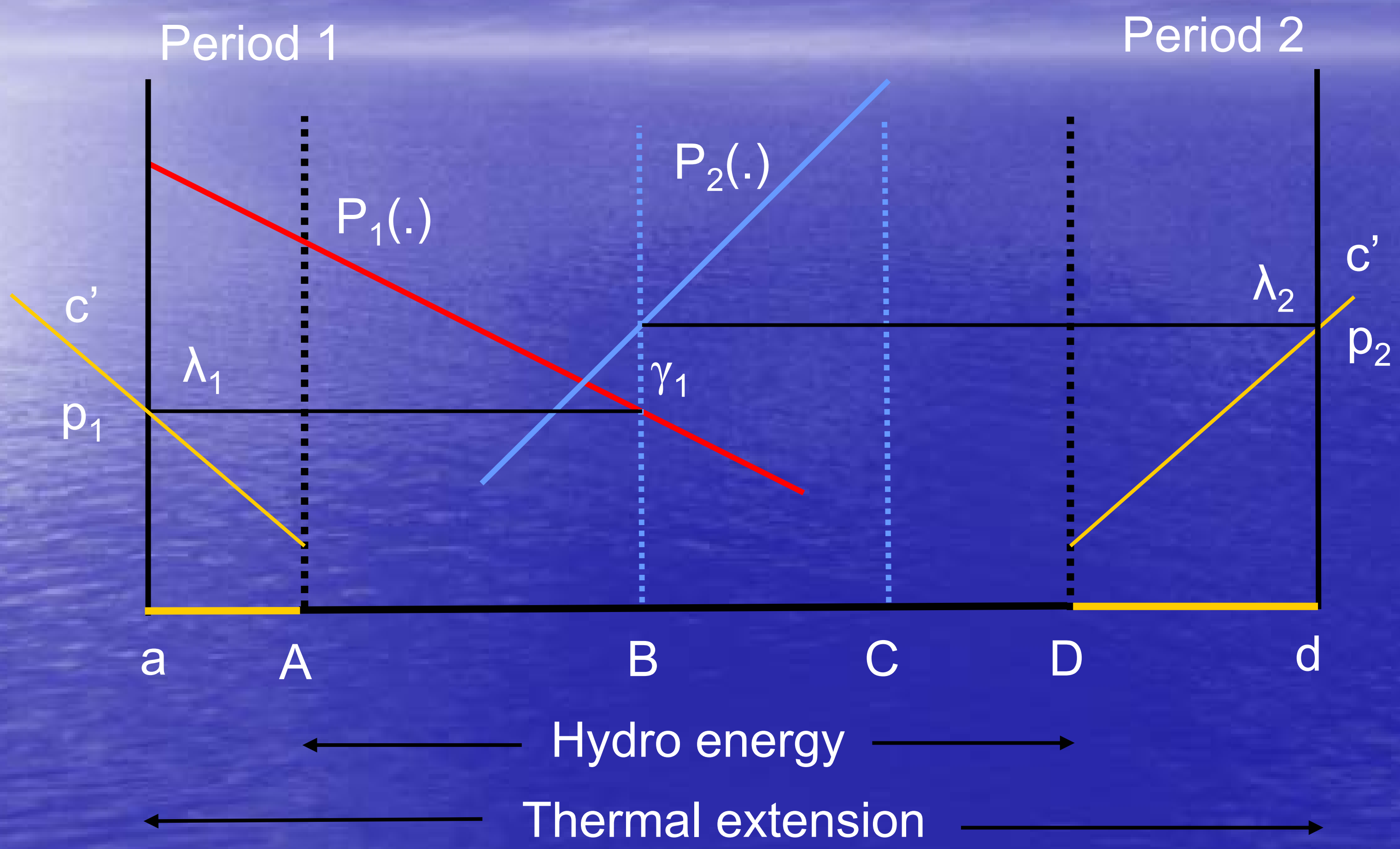
$$p_t(x_t) = v_t = \lambda = c'(e_t^T) + \theta_t$$

- Law of one price

Two periods: extended bath-tub



Thermally extended bath-tub with reservoir constraint



Monopoly with hydro and thermal capacity

$$Max \quad \left\{ \sum_{t \in T} [(p_t(x_t)x_t - c(e_t^T)] \right\} s.t.$$

$$x_t = e_t^H + e_t^T, \;\; \sum_{t \in T} e_t^H \leq W, \; e_t^T \leq \overline{e}^T$$

- Necessary conditions

$$p_t'(x_t)x_t + p_t(x_t) - \nu_t \leq 0 \perp x_t \geq 0$$

$$\nu_t - \lambda \leq 0 \perp e_t^H \geq 0$$

$$-c'(e_t^T) + \nu_t - \theta_t \leq 0 \perp e_t^T \geq 0$$

Monopoly, cont.

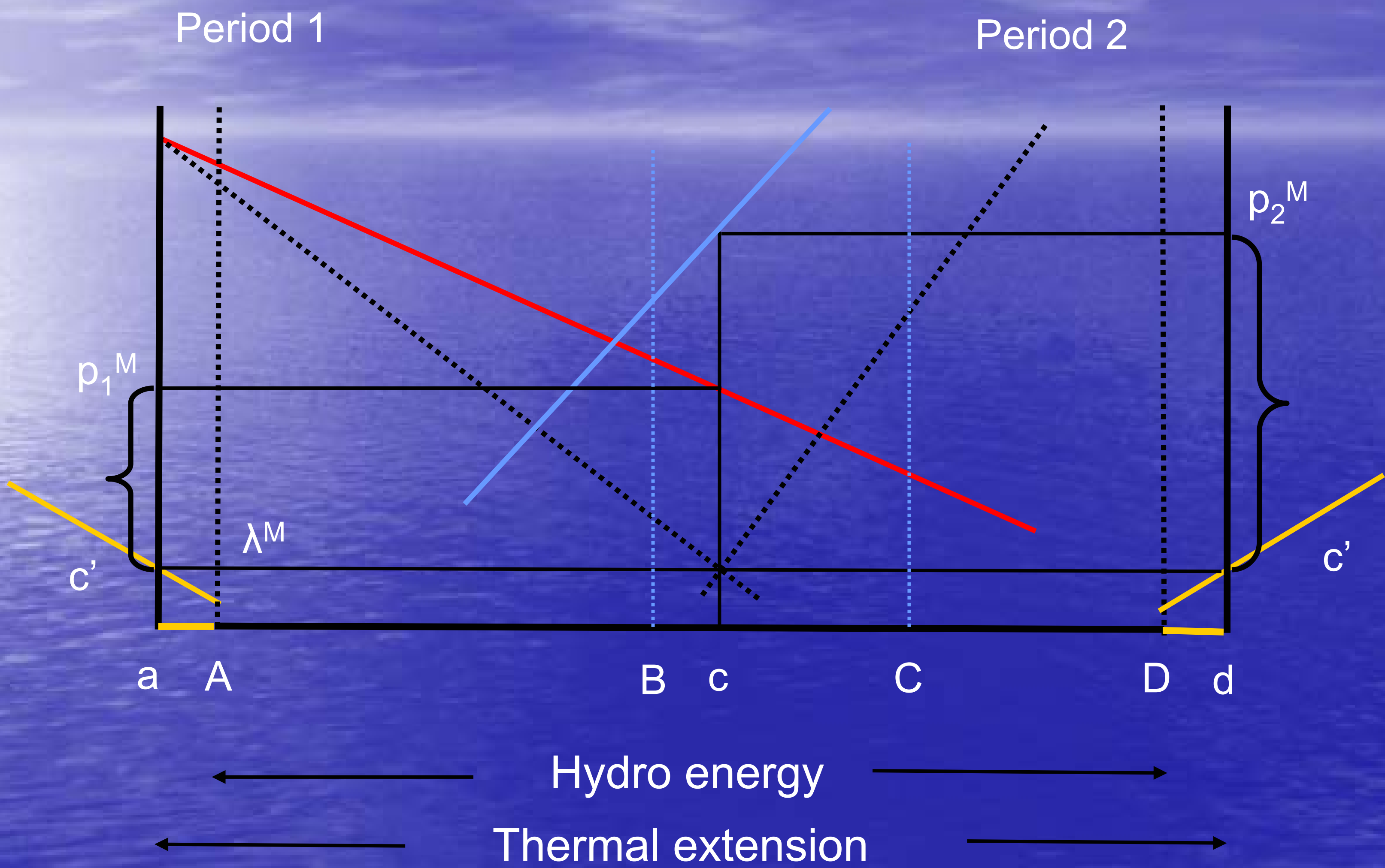
- Constant thermal baseload

$$\lambda = c'(z_t^T) + \theta_t$$

- Law of variable price

$$p_t(x_t)(1 + \breve{\eta}_t) = v_t = \lambda$$

Monopoly and extended bath-tub



Extentions

- Uncertainty
  - Future water values become stochastic variables, system must avoid going dry
- Competitive fringe
  - Hydro or thermal; will reduce possibilities of strategic shifting of water
- Oligopoly game between hydro producers
  - Essentially a dynamic game, reduces the possibilities of strategic shifting of water

Conclusions for the Real World

- A number of physical constraints on hydro producers resulting in fluctuating prices
- Must know the expected water values and thermal supply function in detail in order to identify use of market power
- Quite complex in the case of uncertainty about inflows to reservoirs and temperature dependent demand
- Quite complex to find solutions to dynamic gaming